# Milling Dynamics: Part II. Dynamics of a SPEX Mill and a One-Dimensional Mill

D. MAURICE and T.H. COURTNEY

The dynamics of mechanical milling can be described in either local or global terms. Local descriptions consider aspects of powder deformation, fracture, and welding that transpire when powder is entrapped between grinding media during a typical media collision. Global descriptions consider heterogeneous aspects of milling in specific grinding devices; for example, the distribution of impact energies and powder segregation within a mill. In part I of this series, we described facets of attritor global dynamics. In this article, we do the same for a SPEX\* mill and for a hypothetical one-

\*SPEX is a trademark of SPEX Industries, Edison, NJ.

dimensional vibratory mill. In a following companion article, we compare the dynamics of an attritor with a SPEX mill and, by extension, other kinds of mills. Specific attention is paid to the relative efficiencies of these devices and the characteristics of powder processed in them.

## I. INTRODUCTION

THE kinetics of mechanical alloying (MA) and the size, microstructural scale, and properties of powder made by this process depend on the mill employed for alloying. We have referred to modeling of specific MA devices as "global modeling." Global modeling considers mill heterogeneities. These may be related to variations in media impact velocity (and, thus, energy imparted to the powder trapped between colliding media) or variations in the powder distribution within a mill. Global modeling is distinguished from "local modeling," which considers details of a single collision, *i.e.*, the extent to which powder deformation, coalescence, and fragmentation take place in powder entrapped between media colliding at a specific velocity.

We have previously reported on the global dynamics of an attritor.<sup>[1]</sup> There we presented arguments for direct impacts effecting MA much more substantially than rolling or sliding events. We also related our observation that the latter, which seem to be the dominant media action in an attritor, are really high number-low relative velocity impacts. As the media action in a SPEX mill is predominantly impaction (although not necessarily "head-on," or direct, impaction), the two mills can be compared with regard to their milling efficiency. In this article, we describe the heterogeneous distribution of collisions within a SPEX mill. Results from this analysis are coupled with experiments of others to argue that even in a SPEX mill, only a small fraction of media collisions is effective for MA. We conclude with an analysis of a hypothetical mill bearing some resemblance to a SPEX mill and other mills used for MA.

## II. BACKGROUND

There have been two noteworthy studies on the global mechanics of a shaker mill such as a SPEX mill. Davis *et al.*<sup>[2]</sup> used precision stroboscopy to reduce the apparent vial velocity. Vial motion was then video recorded and analyzed by a "motion analysis" computer translation system. This converted the analog motion of the vial into digital coordinate displacements and velocities. Results from this study provided a frequency distribution of impact angles (curve noted as "Davis, McDermott, and Koch" in Figure 1) and a frequency distribution of the amount of kinetic energy dissipated during inelastic collisions between grinding media balls. The energy-loss distribution was used to estimate the temperature rise powders may experience during an impact.

Courtney and Maurice<sup>[3]</sup> attempted to explain the distribution in impact frequency observed by Davis *et al.* An approach based on a geometrical solid angle distribution of collisions was first considered. This did not explain the observed distribution nearly as well as one that considered collisions to be biased toward glancing collisions. The latter begins to account for the frequency of ball-ball collisions in a SPEX vial (Figure 1). An estimation of the distribution was also made for the condition of a "one ball" mill (*i.e.*, a mill in which only ball-vial collisions occur). This resulted in a distribution (Figure 1, "SPEX mill" curve) reasonably mimicking that observed even to the relative position and magnitude of the inflections in the distribution.

In theoretical local modeling studies,<sup>[3–6]</sup> we have argued that impacts that take place within a few degrees of a headon collision are most effective in contributing to powder morphological evolution. In their SPEX mill studies, Davis *et al.* found only 0.5 pct of impacts occurred as near headon and, furthermore, only 1.1 pct of all impacts could cause substantial powder heating. These results are somewhat analogous to those found in global attritor studies; that is, only a small fraction of media interactions in either mill eventuates in substantial powder evolution. (In this regard, glancing impacts in a SPEX mill might be thought analogous to rolling/sliding events in an attritor.) This is so, even

D. MAURICE, NRC Research Associate, is with the Albany Research Center, United States Bureau of Mines, Albany, OR 97321. T.H. COURTNEY, Professor and Chair, is with the Department of Metallurgical and Materials Engineering, Michigan Technological University, Houghton, MI 49931.

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Fig. 1—Impact angle distributions. The triangles represent distributions deduced by Davis *et al.* for a SPEX mill.<sup>[2]</sup> Curve (a) represents a distribution expected on the basis of random (solid angle) impaction. Curve (b) represents a distribution biased toward glancing impacts. Curve (c) represents a distribution expected for a SPEX mill in which ball-vial impacts are exclusive. The Davis *et al.* results are reasonably mimicked by an appropriate averaging of curves (b) and (c).<sup>[3]</sup>



Fig. 2—Energy loss spectrum deduced by Ref. 2 for inelastic collisions in a SPEX mill absent powder. Most collisions result in an energy loss between  $10^{-3}$  and  $10^{-2}$  J. Very few collisions result in an energy loss approaching 1 J, approximately correlating to a head-on, high velocity impact.

though "substantial" might have a different qualitative implication when comparing the devices. It is also the case notwithstanding that a SPEX mill, as a result of the higher velocity of the balls, is more "efficient" in terms of the processing time required to produce a desired powder structure.

Here we extend the Davis *et al.* description of a SPEX mill. Using the Davis *et al.* results along with simplified mechanics and some reasonable assumptions, we deduce an approximate energy-loss distribution for collisions in a SPEX mill. The results obtained are consistent with the preceding ideas in that only a small percentage of collisions in a SPEX mill result in reasonable energy dissipation.

#### III. GLOBAL DYNAMICS OF A SPEX MILL

The energy-loss spectrum deduced by Davis *et al.* is illustrated in Figure 2. Davis *et al.* also measured the coef-

ficient of restitution (e) of the balls. A value of e < 1indicates energy is dissipated during impact, and for balls not coated with powder, the energy loss is manifested in thermoelastic heat. In the following analysis, we assume that this form of energy dissipation correlates with the energy available for plastic deformation of powder during the impact of coated balls. While the mechanics of impaction of coated balls are much more complex than this, the previous assumption seems tenable. And if the results that follow are not as quantitatively accurate as we would like, the dependence of energy loss as a function of impact angle ought to be qualitatively correct.

Consider two balls having the same mass and traveling along the same line with precollision velocities  $u_1$  and  $u_2$ , respectively. Elementary mechanics show that following the collision, the balls, still traveling along the same line, have the velocities

$$v_1 = \frac{1}{2} \left[ u_1 \left( 1 - e \right) + u_2 \left( 1 + e \right) \right]$$
 [1]

and

$$v_2 = \frac{1}{2} \left[ u_1 \left( 1 + e \right) + u_2 \left( 1 - e \right) \right]$$
 [2]

The kinetic energy lost by each ball is  $[m/2][u_i^2 - v_i^2]$  (i = 1, 2 and m = ball mass). The total kinetic energy loss for the collision is

$$\Delta KE = (m/4) [1 - e^2] [\Delta u]^2$$
 [3]

where  $\Delta u = u_1 - u_2$ . An upper limit on the fractional kinetic energy loss per collision holds when  $u_2 = 0$ , for which case

$$\Delta KE/(KE)_0 = [1 - e^2]/2$$
 [4]

where  $(KE)_0$  is the precollision kinetic energy of ball "1." For e = 0.7 (Davis *et al.* measured e = 0.712),  $\Delta KE/(KE)_0 = 0.255$ . The analysis in Appendix A indicates that for a glancing collision at angle  $\theta$  ( $\theta = 0$  deg corresponding to a head-on collision),

$$\Delta KE = (m/4) [1 - e^2] [u_1 \cos \theta - u_2]^2$$
 [5]

and the maximum fractional kinetic energy loss (when  $u_2 = 0$ ) is

$$\Delta KE/(KE)_0 = 0.5 \ [1 \ -e^2] \cos^2 \theta$$
 [6]

The kinetic energy-loss distribution is related to the impact angle distribution by

$$P(E)\Delta E = P(\theta)\Delta\theta$$
 [7]

where P(E) is the fraction of collisions resulting in an energy loss between KE and  $KE + \Delta KE$  and  $P(\theta)$  is the analogous impact angle distribution expression. The results of converting the impact angle distributions of Figure 1 to energy-loss distributions (*via* Eqs. [6] and [7]) are given in Figure 3. The majority (about two-thirds) of collisions result in modest amounts of energy loss (*i.e.*, < 0.2  $\Delta KE_{\text{max}}$  or 0.05 (=0.255 × 0.2) (*KE*)<sub>0</sub>). In contrast, only from 4 to 7 pct (depending upon the impact angle distribution) of collisions result in appreciable energy transfer (*i.e.*, 0.8  $\Delta KE_{\text{max}}$  or 0.2 (*KE*)<sub>0</sub>).

To put this in the context of Davis et al., we first estimate



Fig. 3—Energy loss distributions for inelastic grinding media collisions for the impact angle distributions shown in Fig. 2.  $\Delta KE_{max}$  corresponds to the maximum possible kinetic energy loss (a fraction of the precollision kinetic energy, discussed in the text). Irrespective of the impact angle distribution, very few collisions result in appreciable energy loss.

the energy above which 4 to 7 pct of the impacts result in an energy loss greater than a specified energy. This is done crudely, assuming a linear distribution in energy loss for energies between  $10^{-3}$  and  $10^{-2}$  J (Figure 2). The threshold energy loss is 9.6 (±0.1) ×  $10^{-3}$  J; *i.e.*, about 5 pct of all collisions result in energy losses exceeding this threshold. Equating this to 0.8  $\Delta KE_{max}$  (with  $\Delta KE_{max} = 0.255$  (*KE*)<sub>0</sub>) results in a corresponding collision velocity of 6.9 m/s. We also note that the "tail end" of the energy-loss spectrum (*i.e.*,  $\Delta KE < 10^{-3}$  J) corresponds to an impact velocity on the order of 2 m/s. By comparison, previous estimates of ball collision velocities in a SPEX mill<sup>(4)</sup> have been on the order of 4 m/s, intermediate between the high and low benchmarks.

Local modeling results are now used to show that the previous deductions are consistent with experiment. In particular, we determine the approximate velocity and frequency of effective impacts in a SPEX mill. Benjamin and Volin<sup>[7]</sup> studied MA of Fe and Cr in such a mill. The balls they used were of the same size as those used by Davis *et al.*, and we assume both sets of balls to have had the same coefficient of restitution. The number of balls used was different in the two studies. However, Davis *et al.* found the energy loss distribution was essentially independent of the number of balls in the mill (for up to 15 balls, the maximum number investigated). We assume the larger number of balls (22) used by Benjamin and Volin does not appreciably alter the impact angle and energy loss distributions.\*

\*We note that for both situations, the balls occupy only a small fraction of the vial volume; about 5.4 and 7.9 pct, respectively.

Benjamin and Volin<sup>[7]</sup> determined powder hardness and interlamellar (phase) spacing as a function of milling time. If the collision frequency is known, knowledge of the interlamellar spacing permits estimation of the strain per collision. This, in turn, allows estimation of the plastic constitutive law of the powder. These empirical data can be incorporated into a computational program (MAP2) apropos to local modeling.<sup>[6]</sup> The program determines particle hardness, size, and other parameters as a function of the

 
 Table I. Impact Characteristics Required to Match Hardness of Mechanically Alloyed Fe-Cr Alloys

Milling Time (Minutes)	Powder Hardness (kg <sub>f</sub> /mm <sup>2</sup> )	Number of Impacts / Number per Minute*	
		v = 2  m/s	v = 4  m/s
0	170	_	
20	230	8/2.5	4/5.0
40	300	19/2.1	9/4.4
60	360	30/2.0	14/4.3

\*The number of impacts required to match the hardness is the number per minute multiplied by the milling time.

number of impacts the powder experiences. Although the program considers only an "average" particle, results can be used to estimate kinetic aspects of the process for comparison to the previous deductions.

First, consider the low end of the velocity spectrum (2 m/s). Davis *et al.* measured approximately 1800 collisions per second with 15 balls; if we assume them to be ball-ball collisions, then each ball participates in about 240 collisions per second. Each impact involves only a small fraction of the powder associated with a ball, so it will take a number of impacts by a given ball before we can be confident that all particles associated with it have, on average, been struck once. The required number of such impacts<sup>[6]</sup> is

$$V_{p}/V_{c} = [2D_{b}/CRh_{0}v\rho_{p}] [\rho_{b}H_{\nu}/3]^{1/2}$$
[8]

where  $V_p$  is the powder volume "associated" with each ball,  $V_c$  the powder volume influenced by the collision (the collision volume),  $D_b$  and  $\rho_b$  the ball diameter and density,  $H_v$  the powder hardness, *CR* the charge ratio,  $\rho_p$  the (bulk) powder density,  $h_0$  the powder-coating thickness, and v the impact velocity. Using Eq. [8], we find that for v = 2 m/s, the required number of impacts by a ball to ensure that each particle is struck once is 4550. At 240 impacts per second, the associated milling time is 19 seconds. Were a velocity of 4 m/s used in the preceding exercise, this time would be 9.5 seconds.

Using MAP2, we find that for v = 2 m/s, an "effective" impact is required roughly every 2.2 min (132 s; Table I) in order to match the powder hardness-milling time behavior observed by Benjamin and Volin.<sup>[7]</sup> Thus, at best, 1 in 6.95 (the ratio of 19 to 132 seconds) impacts are deforming the powder. If a velocity of 4 m/s is chosen, an effective impact is needed approximately every 4.5 minutes (Table I); thus only about 1 in 28.4 impacts is useful in this case. And while the MAP2 program (and the model on which it is based) does not correctly predict mechanical response at high relative velocities, using Eq. [8] and coarsely extrapolating on the basis of Table I suggests that for v = 6.9m/s, the number of effective impacts is roughly consistent with the percentages (0.4 to 1.1 pct) of impacts considered effective by Davis et al. They also are consistent with the preceding description of energy-loss distributions, at least when taking into account the approximate nature of these exercises.

#### **IV. A ONE-DIMENSIONAL MILL**

As noted previously, a full understanding of the mechanics of MA necessitates incorporation of the local me-



Fig. 4—A one-dimensional shaker mill. The mill, of length L and diameter D, contains a varying number of balls also of diameter D. The balls are presumed coated with powder. The mill is oscillated at a frequency  $\omega$  through an amplitude A.

 Table II.
 Standard Parameters Used in MAP3 Simulations

 Except When Noted Otherwise

5.7 cm
17.0/s*
5.7 cm
6.4 mm / 3/0.34
3.2 mm/7/0.39
12.7 mm/1/0.22
7.8 g/cm <sup>3</sup>
200 µm
10.0 s

\*Edwards and Maurice,<sup>[9]</sup> using stroboscopic lighting techniques, found that a 60 Hz power source results in a SPEX mill making 1020 oscillations per minute, *i.e.*, about 17 oscillations per second. The frequency,  $\omega$ , is about 107/s =  $2\pi \times 17$ . For simulations at 50 Hz, a frequency of 5/6 that used for 60 Hz was employed.



Fig. 5—Minimum, maximum, and mean media collision velocities, as determined by MAP3, for the one-dimensional mill. (Mill operating conditions are specified in Table II; i.e., three 6.4-mm balls corresponding to a fill factor of 0.34.) A transient of about 2 s is followed by an approximately steady-state milling condition.

chanics of an individual grinding media collision<sup>[3-6]</sup> within the framework of the dynamics pertaining to a specific mill.<sup>[1]</sup> We have attempted to do this heretofore by investigating the global mechanics of such a specific mill and extracting from these studies an educated guess as to the appropriate collision velocity, frequency, *etc.* to be used in the local analysis. It would be worthwhile to model global mechanics within a framework that utilizes aspects of local modeling. Here, we attempt to do so in a simplified way. We note that this effort differs from previous global-modeling studies<sup>[2,8]</sup> in that the influence of the powder on the behavior of the media or the kinetics of media interaction is incorporated.

We model a one-dimensional mill (Figure 4). On this account, it bears little resemblance to most conventional mills used for mechanical alloving which involve media motion in three dimensions. For example, in a SPEX mill the diameter of the mill is usually much greater than that of the grinding media balls within it. This gives rise to the large number of (relatively ineffective) glancing impacts characteristic of this device. The one-dimensional mill we consider here results in only direct impacts. It can be considered somewhat analogous to a SPEX mill in which glancing impacts are disregarded. As will be seen, we are able to extract considerable insight into the effects of milling parameters without the need for greater mathematical or computing complexity. The computer program we have developed to describe this mill (discussed subsequently) might be of use in mill optimization in much the same way the programs we have developed to describe local milling mechanics are useful for investigating the effect of material properties on the kinetics of MA.[6.10]

As shown in Figure 4, a vial of length L and diameter D contains balls, also of diameter D. The balls are presumed coated to a thickness  $h_0$  with powder of hardness  $H_v$ . The vial is then agitated at a frequency  $\omega$  through an amplitude A. The positions of the two vial ends at any time are given by

$$x = \pm L/2 + A \sin(\omega t)$$
 [9]

and the mill velocity is

$$v = \omega A \cos (\omega t)$$
 [10]

The balls are considered to start at rest against one end of the vial. The positions of their centers are given by

$$x_i = -L/2 + (i - 1/2) D$$
 [11]

where *i* is the ball number. The vial is accelerated during the first quarter of the first oscillation, following which point it begins to decelerate. The balls then leave the vial end and (presuming no frictional sliding losses) continue at their initial velocity ( $\omega A$ ) until they strike the other end of the vial. The foremost ball (adjacent to the end of the vial at impact) will be deformed the greatest amount. Hence, it will have the lowest velocity upon leaving the second vial end, while the hindmost ball will have the greatest velocity. During each collision, the energy that goes into plastic deformation of the powder can be calculated using the method described in Reference 5. A coefficient of restitution\* is

determined for each collision:

$$e = (1 - (U_p/U_E)^{1/2}$$
[12]

where  $U_p$  is the plastic deformation energy imparted to the powder<sup>[5]</sup> and  $U_E$  the precollision kinetic energy. It should be noted that the coefficient of restitution is calculated in-

<sup>\*</sup>The coefficient of restitution is defined as the ratio of the pre- to postcollision velocity. Let these velocities be v and u, respectively, with corresponding kinetic energies  $U_{E}$  and  $U'_{E}$ . Then  $U'_{E} = U_{E} - U_{\rho} = U_{E} (1 - U_{\mu}/U_{F})$ . Defining e as the ratio u/v leads directly to Eq. [12].



Fig. 6—Distribution of steady-state media collision velocities for the one-dimensional mill as ascertained through MAP3. The distribution in collision velocity is not continuous; the effect is most marked when a few numbers of balls are employed. For example, there are two "spikes" in the collision velocity when one large (12.7-mm diameter) ball is employed. When seven small balls (3.2-mm diameter) are employed the collision velocity distribution approaches a continuum. Constant vial length is used in the simulation. Mill parameters are listed in Table II.

dependently for each ball in the computational scheme described subsequently. And because thermoelastic energy dissipation is not accounted for, the previous expression yields an upper limit on e and, accordingly, a maximum total postcollision kinetic energy for the balls involved.

The separation velocities of the balls are given by Eqs. [1] and [2]. In a ball-vial end collision, it is assumed there is no change in the velocity of the vial. The balls are assumed to maintain their separation velocities until the next impact. With repeated traversals of the length of the vial, "steady-state milling" is attained.

The physics we have described have been incorporated into a computational program (MAP3) capable of tracking the events noted. In the computational simulation, each collision is recorded along with the relative velocity of the impacting balls (or ball if it strikes a vial end) and the plastic deformation energy expended. Instructions for acquiring a copy of MAP3 are given in Appendix B. Before describing some predictions of MAP3, note that Table II provides a summary of some standard values used in the simulations described for those variables held constant during a given simulation. The simulations described hereafter were all carried out with an approximately constant vial filling factor, f, defined as f = nD/L, where n is the number of balls in the vial. In the simulations, we took f to be 0.4; the program automatically selects the largest integer number of balls that can be used without exceeding this number. Specific values for f and n for each investigation are also provided in Table II.

#### A. The Initial Transient

Figure 5 shows the minimum, maximum, and mean collision velocities determined by MAP3 during the first 10 seconds of milling. A transient period of approximately a 2-second duration is found, after which the relative velocities and the velocity distribution attain a steady state. The simulation illustrated in Figure 5 was conducted with three 6.4-mm-diameter balls. A similar simulation using seven 3.2-mm-diameter balls demonstrated a transient period of about 5 seconds. Clearly, the transient period is short relative to milling times employed practice.

#### B. Collision Velocity Distribution

In Section III, we assumed that the velocity distribution is a continuum. Predictions of MAP3 suggest, however, that this need not always be the case for a one-dimensional mill.



Fig. 7—Average impact velocity as a function of vial length (vial diameter being kept constant in the simulation). Open marks correspond to an excitation frequency of 60 Hz and solid marks to a frequency of 50 Hz. As discussed in the text, some of the behavior manifested in this figure is not intuitive.



Fig. 8—Average collision frequency (per ball) as a function of vial length. The parameters are the same as for Figure 7. Again, the observed behavior is not intuitive.

Table III.Parameters Used in Constructing<br/>Figures 7 and 8

Vial Length (cm)	Number of Balls	Filling Factor
3.7	2	0.341
4.2	2	0.300
4.7	2	0.268
5.2	3	0.363
5.7	3	0.332
6.2	3	0.305
6.7	4	0.376
7.2	4	0.350
7.7	4	0.327

As shown in Figure 6, following attainment of steady-state milling conditions, there are discrete "bands" of relative collision velocities. The number of these bands depends upon the ball size and, therefore, number of balls in the mill (Table II). For example, simulation with one large

(12.7-mm-diameter) ball yields two spikes in the collision velocity distribution; milling with three 6.4-mm balls yields bands at about 0.5, 3.8, 6.2, and 8.0 m/s. As the number of balls is further increased, the collision velocity distribution approaches a continuum, as demonstrated by the use of seven balls of 3.2-mm diameter. A larger number of balls results in more ball-ball collisions, and hence more even distribution of collision velocities.

## C. Influence of Vial Length

Figures 7 and 8 illustrate the effect of the vial length on average impact velocity and collision frequency. These calculations were done using an approximately constant fill factor (see Table III for details of parameters pertinent to Figures 7 and 8), so that the number of balls used in the simulations changes periodically as shown in the figures. We have no detailed explanation for some of the trends exhibited in these figures as they are not all intuitive. For example, as might be expected, relative impact velocity increases with vial length for the two-ball and four-ball situations of Figure 7, but velocity is approximately independent of length for the three-ball situation. Similarly, collision frequency (per ball) decreases with vial length (as might be intuited) for the two- and four-ball situations, but not for the three-ball one. We also note that "discontinuities" in frequency take place each time the ball number increases. Since the frequency is expressed on a per ball basis, this is also not an intuitive result.

The collision velocities and frequencies of Figures 7 and 8 are illustrated for two different excitation frequencies corresponding to AC frequencies of 50 and 60 Hz. To an excellent approximation, the collision velocities and frequencies correlate directly with the frequency of the excitation voltage. For example, the collision frequencies at the two voltage frequencies are in the ratio of  $0.83 \pm 0.02$ . The corresponding ratio for the velocities is  $0.84 \pm 0.015$ .

We have illustrated only some of the mill variables that can be studied with MAP3. Amplitude, fill factor, charge ratio, frequency of oscillation, *etc.* are others that can be investigated. While the one-dimensional nature of the mill on which the program is based precludes direct application to conventional mills, it may be useful for systematically studying the effect of alterations in machine design and operating parameters on device efficiency. As noted, MAP3 has two advantages that many other models do not. First, it considers multiple media interactions; and, second, it incorporates the effect of the powder charge on the media dynamics.

## V. SUMMARY

In this article, we have described some aspects of the global mechanics of two devices, a SPEX mill and a hypothetical one-dimensional mill, commonly used for MA. We have shown, using the work of others and some of our own analysis, that a SPEX mill is an inefficient device, at least in terms of the fraction of media collisions that eventuate in appreciable effects on the powder charge. We have studied the one-dimensional mill for it provides a simplified geometry for incorporating local and global modeling. The program that has come from this study, MAP3, may be of

some use to device designers, for machine geometry and characteristics can be varied in this program. Such an effort complements other efforts aimed at integrating global and local modeling and that focus on the effect of material properties on the kinetics of alloying as described in the article following.

## APPENDIX A

The description of an impact taking place at an angle  $\theta$  proceeds similarly to that for a head-on collision. Ball 2 initially translates in the x direction with velocity  $u_2$ . Ball 1 has two components of velocity  $u_{1x}$  and  $u_{1y}$ , and the collision angle  $\theta$  is defined by  $\cos \theta = u_{1x}/u_1$ , where  $u_1$  is the magnitude of ball 1's velocity. The y velocity component is presumed unchanged during collision (so as to conserve momentum). The analysis is the same as that described in the text, except for the following alterations: in Eq. [1],  $v_{1x}$  substitutes for  $v_1$  and  $u_{1x}$  substitutes for  $u_1$ ; in Eq. [2],  $u_{1x}$  substitutes for  $u_1$ .

## APPENDIX B Access to programs

The MAP2 and MAP3 program packages are available by electronic transfer. Those with access to the Internet can obtain a copy *via* remote ftp. To do so, follow this procedure:

connect to ftp.mm.mtu.edu

at the log in prompt, type "ftp"

password: provide your full e-mail address (e.g., thc©mtu.edu)

You will be informed that you have successfully logged in. At this point, type in

"cd pub/MAP"

Take the MAP2 (or MAP3) file by the command

"get MAP2.README" (or "get MAP3.README") "get MAP2.tar" (or "get MAP3.tar")

This will put the files in your computer. The README files give instructions for setting up the programs. A TUTORIAL file found in both of the .tar packages gives specific instructions for running the program. A tar file is an archive containing one or more other files in a convenient package. Use "tar xf MAP2.tar" to extract those files from the archive for MAP2; the same procedure applies for MAP3.

Those lacking Internet services may acquire a copy via modem dial up to MatChat by following these instructions. To access MatChat, call +510-655-1753 using your modem set at  $\frac{8}{1}$  (*i.e.*, 8 data bits, no parity, and 1 stop bit); this is the standard setting for calling all dial-up bulletin board systems, but differs from the setting for calling most mainframes. MatChat accepts speeds up to 14,400 bps (baud), so the modem should be set at the highest speed. Once connected, follow the prompts until the "Main Area Function?" prompt is reached. Then the D command takes vou to the Files Area, where D downloads a file and U lets a file be uploaded to MatChat to make it available for others (Syscop will first scan it for viruses, etc.). First-time callers are requested to leave a message introducing themselves and should download the DIR.ZIP file, which describes each file in the file library. Do this using the command F;D;DIR.ZIP. If the terms "download" and "ZIP" are not familiar, read the Bulletins from the main menu. Regardless of transfer protocol used, both the .README and .tar files should be taken.

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